Name: ____ Date: ___________________

Water Turbine Worksheet Answers

Problem

What is the problem that you are trying to solve with the design of a water turbine for a house?

Answers will vary. They should include how to generate electricity for a house using a renewable energy source.

Background Knowledge

🡐See the activity's Introduction/Motivation section for answers.

What are some things that you already know about water turbines?

1.
2.
3.

Possible answers:

- **Water strikes the turbine blades and spins the turbine, which is connected to a generator with a shaft**
- **The electricity generated from turbines is sent through power lines of the power grid to where it is needed**
- **Not all the water's energy transfers into electrical energy; during the transfer, some energy is lost through friction**
- **Water is a considered a renewable energy source**

Brainstorming Ideas

Before building a water turbine, engineers brainstorm many ideas for the design. Many different designs exist for water turbines. Some use blades shaped like a fan's blade with a curvature; some might be flat; some are shaped like buckets. Use the space below to record your group's brainstorming session (ideas, drawings, etc.) for the design of your turbine. Remember no idea or suggestion is "silly."

Design

Use the space below to detail your group's final design for the water turbine. Be specific. Include drawings and dimensions as appropriate. (Note: remember to design your turbine with the blades on the end **opposite** where the hole is drilled in the wood block.)

Testing and Analysis

Collect and record the following data at three separate heights:

	Height 1	Height 2	Height 3
Height of water above turbine			
Mass of water to be poured			
Time water was flowing over blades			
Voltage produced			

1. Calculate the potential energy of the water using $PE = mgh$

2. Calculate the water's final velocity just before hitting the turbine blades using

   $$PE = KE = \frac{1}{2}mv^2$$

   Students should solve for $v = \sqrt{2gh}$

3. Calculate the mass flow rate ($\dot{m} = \frac{m}{t}$).

4. Calculate the theoretical power your turbine should generate using the formula $P = \dot{m}gh$ at each height.

5. What can you conclude about the voltage produced as related to the height of the water?

   The voltage produced by the motor should increase as the water falls from locations higher above the turbine.

Evaluation/Questions

1. Most water turbines in use today are able to achieve efficiency ratings of at least 90%. Identify three causes of poor efficiency in your turbine design, that is: what are some reasons for why the efficiency of your (or any turbine) could never be 100%? Can you think of any ways to fix some of these causes for loss in power in your turbine? Why would it be important for engineers to identify where power is being lost and work on ways to reduce these effects?

   Example answer: One reason why an efficiency of 100% can never be obtained by our or any turbine is because of friction between the turbine and the shaft of the motor. These parts will never spin together without some friction and that friction reduces the amount of energy the turbine is able to produce. Another reason is because friction also occurs between the water and the turbine blades. This friction also takes away some of the power production of the turbine as all the potential energy is not able to be converted into electrical energy. A third reason is that the motor (generator) itself is not 100% efficient. The motor is not able to convert 100% of the water's energy into electrical energy, even if the turbine itself was 100% efficient at transferring the water's energy to the shaft of the turbine. It is important for engineers to identify where power is being lost so they can maximize the energy efficiency of the system and save valuable resources and cost.

2. Your engineering firm has been designing an energy-efficient house. The city that the house is located in has been chosen by the government as a site for a small hydroelectric power plant and dam. After analyzing the small river, three possible locations are found to be suitable for the dam and hydroelectric water plant. The city has enlisted your help to determine where to place the dam and hydroelectric power plant. The table below provides specific details of each location. Use this information to help the city determine the best location to place a hydroelectric power plant.

	Water head possible after dam placed (m)	**Average Velocity of flowing water (m/s)**	**Cross-sectional area of water flow (m^2)**	**Cost of building dam ($)**
Location 1	1.75	2.1	7.4	74,000
Location 2	1.9	1.8	8.4	84,000
Location 3	1.5	1.9	7.8	78,000

   a. Using information in the table, calculate the volumetric flow rate, Q, at each location.

$$Q = vA$$

$$Q_1 = 2.1\frac{m}{s} * 7.4\,m^2 = 15.54\,\frac{m^3}{s}$$

$$Q_2 = 1.8\frac{m}{s} * 8.4\,m^2 = 15.12\,\frac{m^3}{s}$$

$$Q_3 = 1.9\frac{m}{s} * 7.8\,m^2 = 14.82\,\frac{m^3}{s}$$

b. The mass flow rate of a flowing fluid can be calculated using the equation $\dot{m} = Q\rho$ where Q is the volumetric flow rate (m^3/s) and ρ is the density of the fluid (the density of water is 1000 kg/m^3). Calculate the mass flow rate ($\dot{m}$) at each location.

$$\dot{m} = Q\rho$$

$$\dot{m}_1 = 15.54 \frac{m^3}{s} * 1000 \frac{kg}{m^3} = 15{,}540 \frac{kg}{s}$$

$$\dot{m}_2 = 15.12 \frac{m^3}{s} * 1000 \frac{kg}{m^3} = 15{,}120 \frac{kg}{s}$$

$$\dot{m}_3 = 14.82 \frac{m^3}{s} * 1000 \frac{kg}{m^3} = 14{,}820\ kg/s$$

c. Calculate the power that the water could theoretically produce at each location.

$P = \dot{m}gh$, where g=9.8 m/s

$$P_1 = 15{,}540 \frac{kg}{s} * 9.8 \frac{m}{s^2} * 1.75\ m = 266{,}511\ W$$

$$P_2 = 15{,}120 \frac{kg}{s} * 9.8 \frac{m}{s^2} * 1.9\ m = 281{,}534\ W$$

$$P_3 = 15{,}120 \frac{kg}{s} * 9.8 \frac{m}{s^2} * 1.5\ m = 217{,}795\ W$$

d. If the turbines to be used at the power plant have an operating efficiency of 91.4% what is the actual power that will be generated at each location?

If η is the turbine efficiency then the actual power generated will be equal to:

$$\eta 1 = 266{,}511\ W * 0.914 = 243{,}591\ W$$

$$\eta 2 = 281{,}534\ W * 0.914 = 257{,}322\ W$$

$$\eta 3 = 217{,}795\ W * 0.914 = 199{,}064\ W$$

e. Calculate how much energy (in kW-hours) this turbine would produce in one year at each location.

$$E = P * t$$

$$1\ year = 365\ days * 24 \frac{hours}{day} = 8760\ hours$$

$$E_1 = P_{1actual} * 8760\ hours = 243.591\ kW * 8760\ h = 2{,}133{,}857\ kWh$$

$$E_2 = P_{2actual} * 8760\ hours = 257.322\ kW * 8760\ h = 2{,}254{,}141\ kWh$$

$$E_3 = P_{3actual} * 8760\ hours = 199.064\ kW * 8760\ h = 1{,}743{,}801\ kWh$$

f. If the hydroelectric power plant takes 150,000 kW-hours a year to operate and this energy is produced at the plant, how much energy would be left over for your neighbors and yourself to use at each location?

$$E_{1usable} = E_1 - 150{,}000\ kWh = 2{,}133{,}857\ kWh - 150{,}000 = 1{,}983{,}857\ kWh$$

$$E_{2usable} = E_2 - 150{,}000\ kWh = 2{,}254{,}141\ kWh - 150{,}000 = 2{,}104{,}141\ kWh$$

$$E_{3usable} = E_3 - 150{,}000\ kWh = 1{,}743{,}801\ kWh - 150{,}000 = 1{,}593{,}801\ kWh$$

g. The government would like to have the plant produce at least 6500kW-hours of energy each year for the town's 300 residents. Based on cost and performance, at which location would your group recommend the dam be built? Explain why.

$6500\ kWh * 300 = 1{,}950{,}000\ kWh$

We recommend building the dam at location 1. This is because the total power needed to be produced by the plant is 1,950,000 kWh. This value is produced at two locations, location 1 and 2. But since it is less expensive to build a dam and power plant at location 1, we recommend building the dam there.

h. Typical coal power plants can produce about 2 kW-hours of energy per kg of coal burned. How much coal must be burned to produce 6500kW-hours of energy for the town's 300 residents? If 1 kg is equal to 2.2 pounds, how many pounds of coal are burned in one year to produce 6500 kW-hours of energy for 300 people?

$$6500\ kWh * 150\ residents = 1{,}950{,}000\ kWh$$

$$\frac{1{,}950{,}000 kWh}{2\ {}^{kWh}/_{kg}} = 975{,}000\ kg * 2.2\frac{lbs}{kg} = 2{,}145{,}000\ lbs\ of\ coal$$

i. Why would you recommend the government to build a hydroelectric dam to power this city? How would the dam affect the individuals and the environment? Write a short persuasion piece to help the government understand the advantages of a hydroelectric dam in this area.