Mechanics of Elastic Materials

Why study mechanics?

- Useful for the analysis and design of load-bearing structures, such as:
  - buildings
  - bridges
  - space shuttles
  - prosthetics
  - biological implants
- Also used to characterize materials

Stress

- The force per unit area, or intensity of the forces distributed over a given section. (units = Pascals [Pa] or pounds per square inch [psi])

$$\sigma = F/A$$

- Stress is how engineers normalize the force that is applied to a material to account for differences in geometry.

- Useful for predicting failure conditions for materials.

Strain

- Deformation per unit length (units: none [unitless])

$$\varepsilon = \Delta L / L$$

- Strain is how engineers normalize the deformation that a material experiences to account for differences in geometry.

- Useful for determining how much a material can deform before failure.

Modulus of Elasticity

- A representation of the stiffness of a material that behaves elastically (units: Pascals [Pa] or pounds per square inch [psi])

$$E = \sigma/\varepsilon$$

- What equation is this similar to?

$$k = F/\Delta x$$

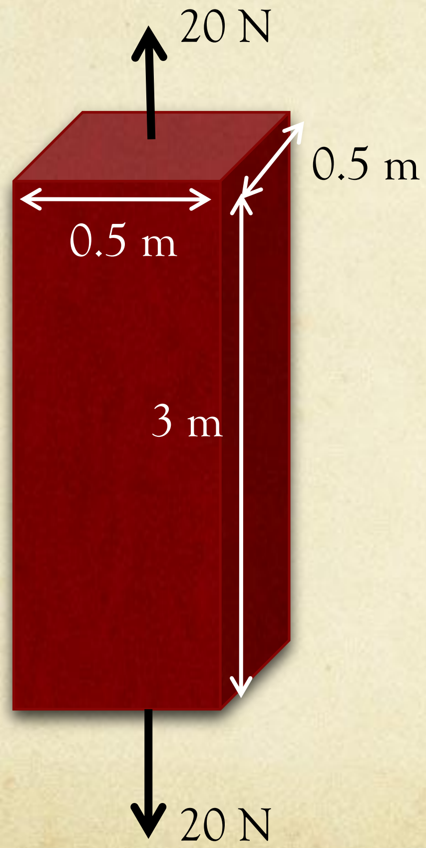
- Modulus of elasticity is how engineers characterize material behavior.
- Useful for knowing how materials behave, material selection for device design, and calculating the stress in a material since it is easier to measure deformation than it is to determine the exact force on a material.

Solid Mechanics

In-Class Examples

Example 1

- This rod is exposed to a tensile force of 20 N. What is the stress in the rod?



$\sigma = F/A$

$F = 20\ N$ (given)

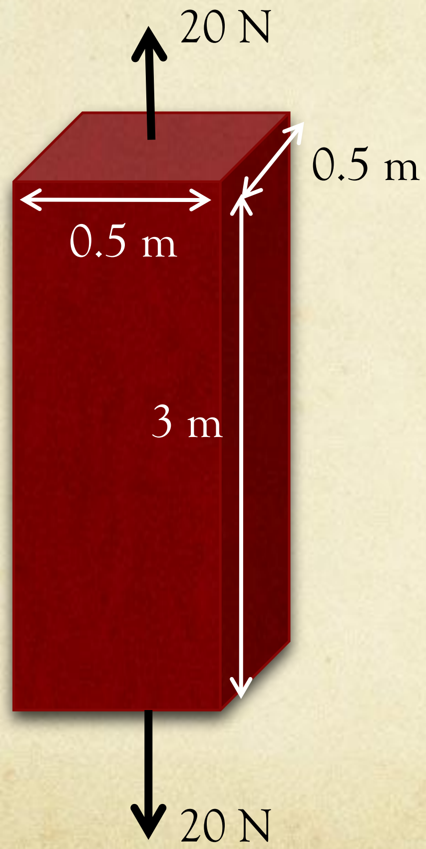
$A = 0.5\ m * 0.5\ m = 0.25\ m^2$

$\sigma = 20\ N / 0.25\ m^2$

$\sigma = 80\ Pa$

Example 2

- The rod below is exposed to a tensile force of 20 N and elongates by 0.03 m. Calculate the strain.



$\varepsilon = \Delta L / L$

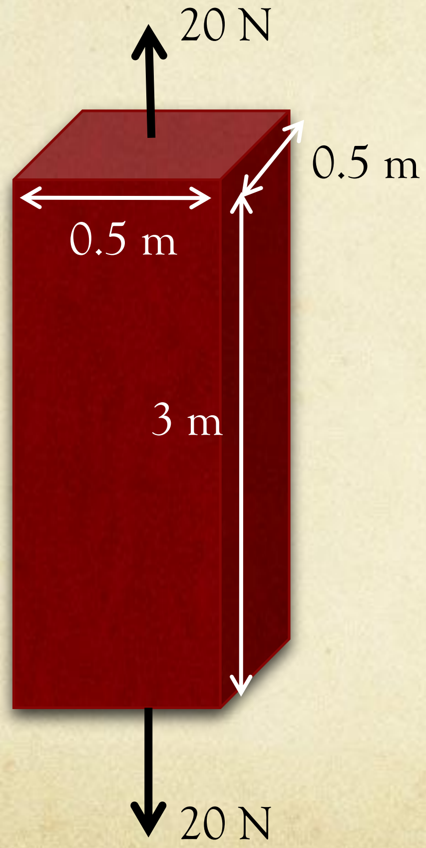
$\Delta L = 0.03$ m (given)
$L = 3$ m

$\varepsilon = 0.03 \text{ m} / 3 \text{ m}$

$\varepsilon = 0.01$

Example 3

- The rod below is exposed to a tensile force of 20 N and elongates by 0.03 m. Calculate the modulus of elasticity.



$E = \sigma/\varepsilon$

σ = 80 Pa (from first example)
ε = 0.01 (from second example)

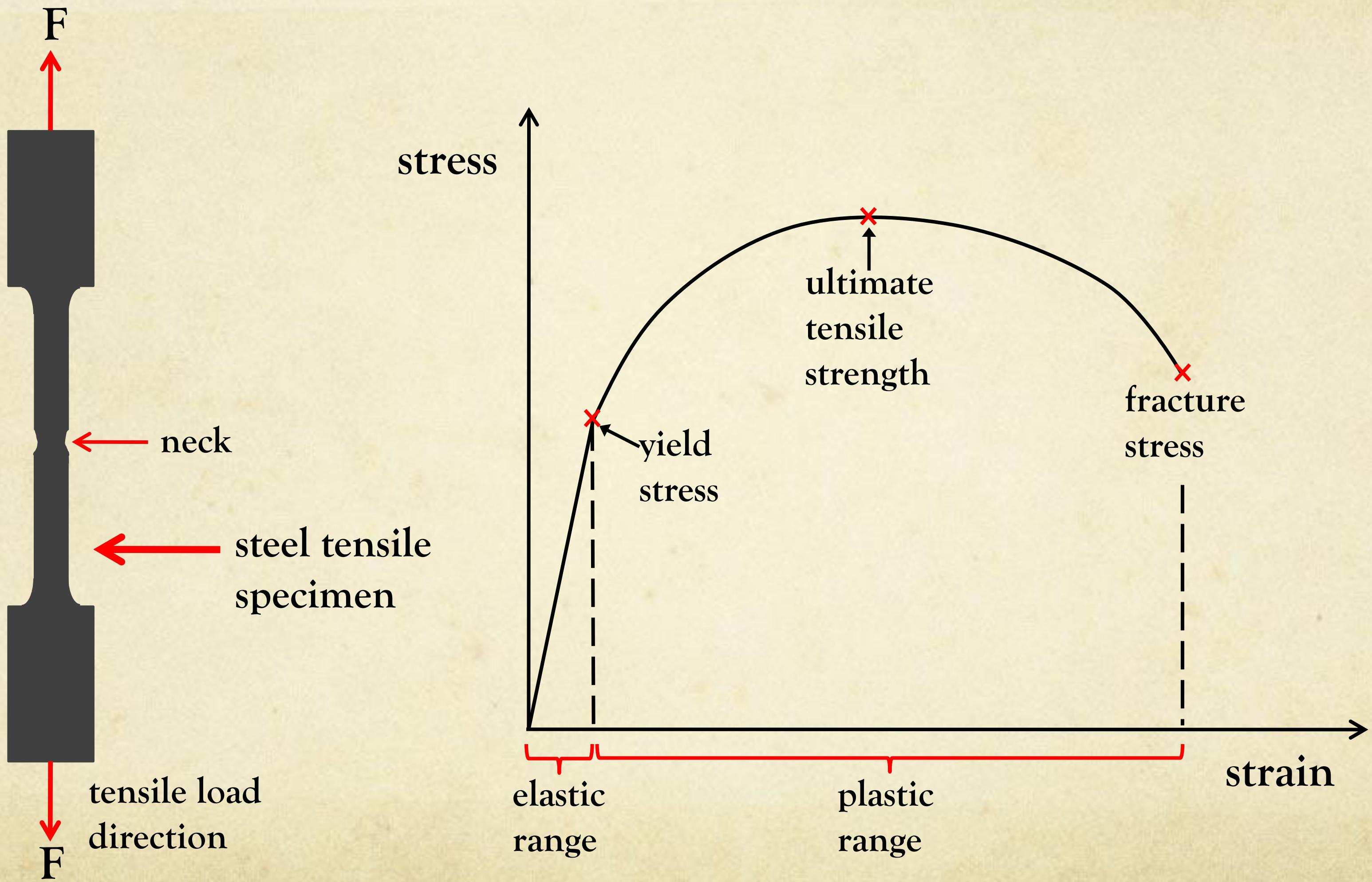
E = 80 Pa / 0.01

E = 8000 Pa or 8 kPa

Next:

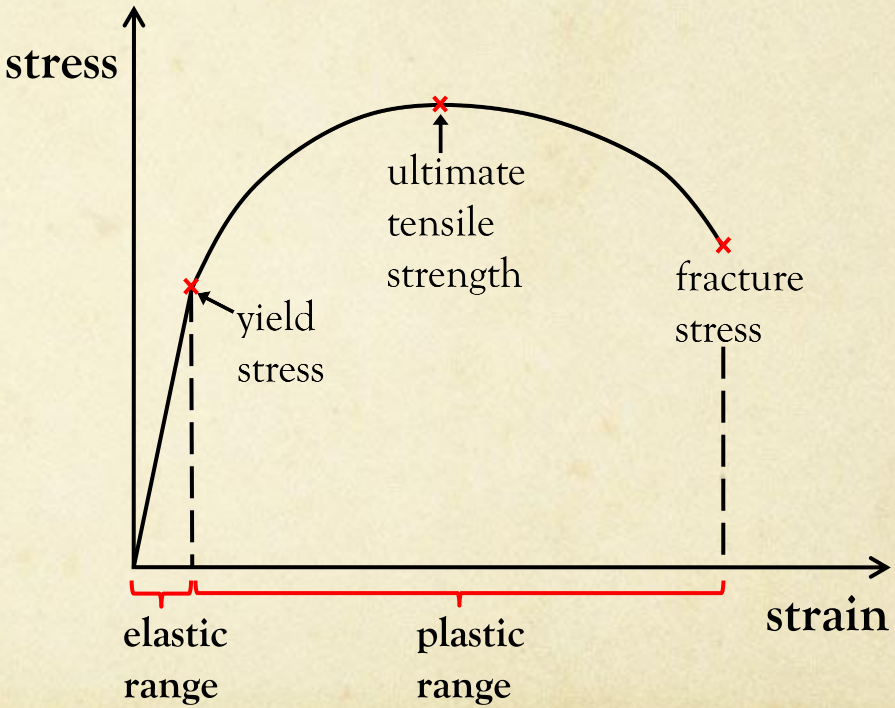
Complete the Solid Mechanics Worksheet

Elastic Behavior



Understanding the Stress-Strain Curve

- **elastic range –** The linear portion of the stress-strain curve. When the force is released, the material returns to its original dimensions.

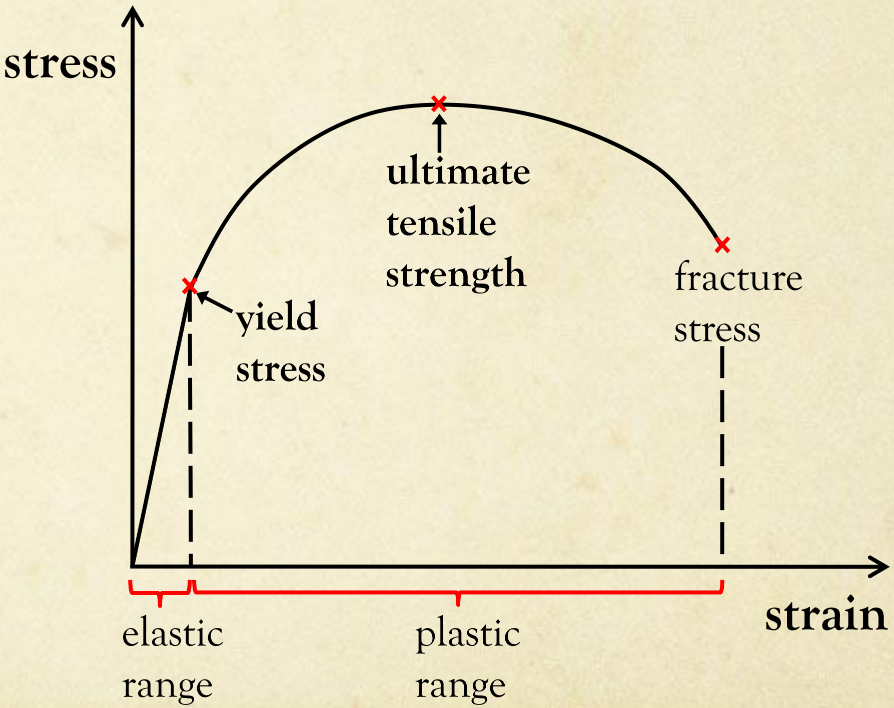


- **plastic range –** The region of permanent deformation.

Understanding the Stress-Strain Curve

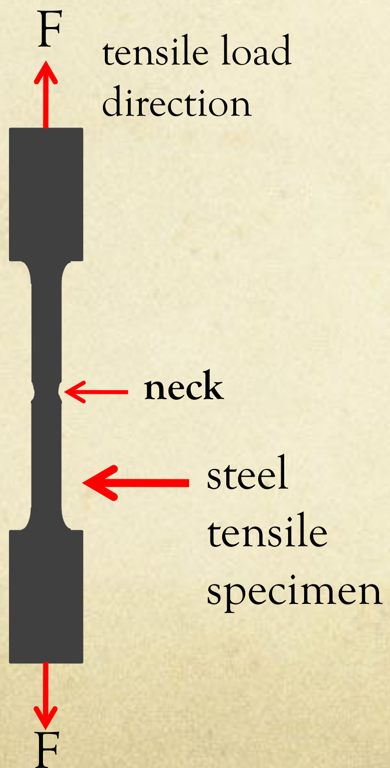
- **yield stress –**
  The minimum stress that causes permanent deformation.

- **ultimate tensile strength –**
  The maximum stress that the material can withstand.
  Also defines the beginning of necking.



The Stress-Strain Curve

- **necking** – A localized decrease in cross sectional area that causes a decrease in stress with an increase in strain.
- **fracture stress** – Stress in which the material fails.



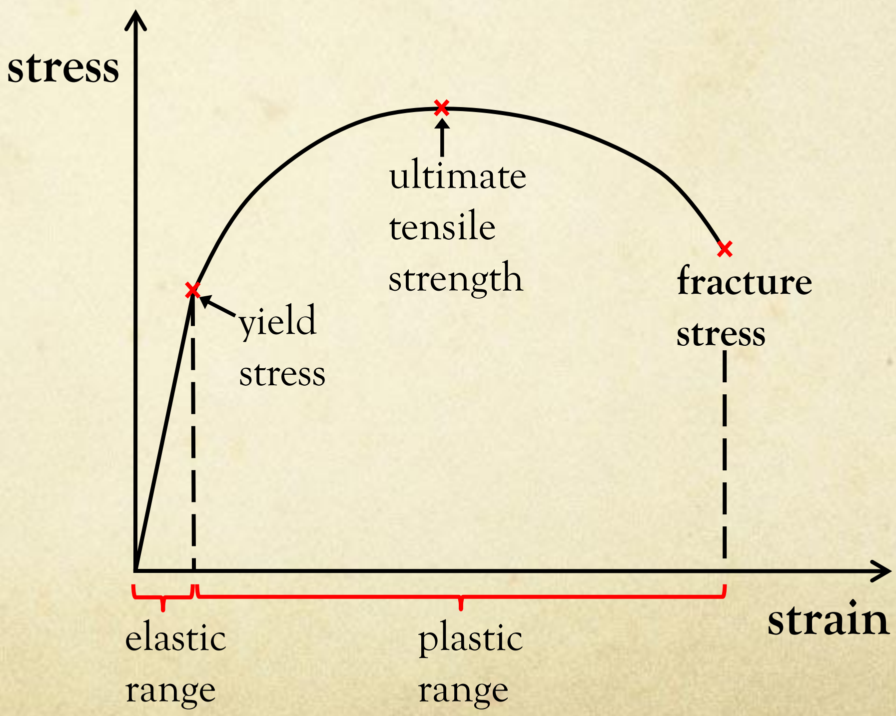


Image Sources

- NOAA http://www.photolib.noaa.gov/htmls/corp2239.htm

- tomruen, wikimedia.org http://sv.wikipedia.org/wiki/Fil:I-35W_bridge_collapse_TLR1.jpg

- Glenn Research Center, NASA http://www.nasa.gov/centers/glenn/moonandmars/med_topic_atomic_oxygen.html